Dynamical phase diagram of a metamagnetic model subject to an oscillating magnetic field

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We investigated the dynamical behavior of an Ising model in a square lattice subject to a time-dependent external magnetic field. In our model the exchange coupling between first neighboring spins in the horizontal direction is different from that of the vertical direction. We have employed the master equation approach for the Glauber stochastic process and used the dynamical pair approximation in order to decouple the hierarchy of equations. We have found the stationary phase diagram for this model in the plane amplitude of the oscillating field versus temperature, for different values of the frequency of the external field and of the ratio between the vertical and the horizontal couplings. Depending on these values, the phase diagram can exhibit the ferromagnetic, paramagnetic, and antiferromagnetic phases. The transition between these phases can be continuous or discontinuous, and the model may also display a tricritical behavior. [S1063-651X(99)01504-4]

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The structure of the phase diagram of a ferromagnetic Ising system is completely modified when an external magnetic field is applied to the system. In fact, when the magnetic field is zero the ferromagnetic Ising model in two dimensions undergoes a continuous phase transition between the ordered and disordered phases at a well defined critical temperature [1]. However, the application of a uniform and constant magnetic field to the system destroys the phase transition and only one phase becomes stable. On the other hand, if the magnetic field is sinusoidally varying in time the system again exhibits a phase transition between the ferromagnetic and paramagnetic phases [2]. Besides, the transition can be continuous or discontinuous depending on the values of the temperature and of the amplitude of the oscillating field. The point connecting the continuous and discontinuous lines is known as the dynamical tricritical point.

However, if the spins of the system are coupled by an antiferromagnetic exchange interaction, the phase transitions are always continuous for both static and varying magnetic fields [3]. In fact, this behavior is observed for small values of the competition between the frequencies of the field and that of spin flipping in the heat bath. Only at very high frequencies of the field does the system remain in a stationary antiferromagnetic state, that is, the transition to the disordered state does not occur.

In this work we have studied an Ising model in the square lattice subject to an oscillating magnetic field. We have taken into account that the exchange interaction between first neighboring spins depends on the direction of the given coupling. In our model the coupling in the horizontal direction has the value J_1 while its value in the vertical direction is taken as J_2 . Depending on the values of the ratio $r=J_2/J_1$ we have obtained, through the master equation formalism and dynamical pair approximation, continuous and first-order phase transitions. The phase diagram that we have found in this pair approximation for the ferromagnetic case (r<0) is very similar to the mean-field one found by Tomé and de Oliveira [2]. On the other hand, when there is a competition

between the horizontal and vertical couplings, the topology of the phase diagram exhibits strong dependence on the values of r>0, that is, for small values of r this metamagnetic model displays continuous and discontinuous phase transitions, while for large values of r only continuous transitions are possible.

Let us start with the following Hamiltonian model:

$$\mathcal{H} = -\sum_{i,j} \sigma_{i,j} [J_1 \sigma_{i+1,j} - J_2 \sigma_{i,j+1} + H(t)], \qquad (1)$$

where $\sigma_{i,j} = \pm 1$ are the spin variables and $H(t) = H_0 \cos wt$ is the magnetic field which is periodically varying in time with frequency w and amplitude H_0 . If this spin system is put in contact with a heat reservoir at temperature *T*, the spin variables $\sigma_{i,j}$ can be considered as stochastic functions of time. The probability $P(\sigma,t)$ of finding the system in the state $\sigma = (\sigma_{1,1}, \ldots, \sigma_{i,j}, \ldots, \sigma_{N,N})$ at time *t* is given by the solution of the following master equation [4]:

$$\frac{d}{dt}P(\sigma,t) = -\sum_{i,j} w_{i,j}(\sigma)P(\sigma,t) + \sum_{i,j} w_{i,j}(\sigma^{i,j})P(\sigma^{i,j},t),$$
(2)

where $w_{i,j}(\sigma)$ is the transition probability, per unit time, from the state σ to the state $\sigma^{i,j}$ when we flip only the spin at site (i,j). Therefore, if the system evolves according to the Glauber prescription [5], the transition probability, per unit time, of flipping the spin at the position (i,j) of the lattice at time *t* can be given by

$$w_{i,j}(\sigma) = \frac{1}{2\tau} \left[1 - \sigma_{i,j} \tanh\left(\frac{1}{k_B T} [J_1(\sigma_{i-1,j} + \sigma_{i+1,j}) - J_2(\sigma_{i,j-1} + \sigma_{i,j+1}) + H(t)] \right) \right], \quad (3)$$

where k_B is Boltzmann's constant and τ is the relaxation time for a single spin.

From the master equation it is possible to show that the average value of any function $g(\sigma)$ can be calculated by the following equation:

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$$\frac{d\langle g(\sigma)\rangle}{dt} = \sum_{i,j} \langle [g(\sigma^{i,j}) - g(\sigma)] w_{i,j}(\sigma) \rangle.$$
(4)

When $g(\sigma) = \sigma_{i,i}$ the equation of motion for the magnetization depends on the correlation function between nearestneighbor pairs of spins. However, if we write the equation of motion for the pair correlation function, new correlations of higher order also appear. In order to decouple this set of equations we have used the dynamical pair approximation [6,7]. In this way, we obtain a closed set of nonlinear equations which give us the magnetization and the pair correlation functions of interest. Although this procedure is not useful for finding the critical exponents, because they are of mean-field type, it improves the mean-field results concerning the determination of the phase diagram, even though the expressions are more complex than the corresponding meanfield ones. For instance, in a recent work [8] we have employed this approximation to find the phase diagram of a layered metamagnetic model in a constant field. We have shown that there is no evidence for the decomposition of the tricritical point into the critical and bicritical endpoints as predicted by the mean-field calculations [9]. The absence of these special points is in agreement with experimental results [10] and with Monte Carlo simulations performed in a related metamagnetic model [11]. When the parameter r < 0, ferromagnetic case, it is necessary to consider only three different equations of motion: one for the evolution of the magnetization and two others for the evolution of the vertical and horizontal pair correlation functions. In the case of competition r > 0, we consider a layered metamagnetic model, where we divide the square lattice into two alternating sublattices 1 and 2 with magnetizations m_1 and m_2 , respectively. Moreover, we need to define three different types of correlation functions: the intersublattice correlation function r_{v} , that represents the correlation between nearest-neighbor spins in the vertical direction, and the intrasublattice correlation functions r_{hi} , which account for the horizontal correlation function in each sublattice, i = 1, 2. In order to evaluate the mean value on the right-hand side of Eq. (4) we write, in the pair approximation, the weighted probability as the product of the probabilities of pairs of spins belonging to a given cluster of spins. Then, taking σ_a as a central spin on the sublattice a surrounded by its first neighbors, the approximated expression for the probability of this cluster is written as

$$P_{a}(\sigma_{a})\prod_{j} \frac{P_{v}(\sigma_{a},\sigma_{j})}{P_{a}(\sigma_{a})}\prod_{i} \frac{P_{ha}(\sigma_{a},\sigma_{i})}{P_{a}(\sigma_{a})},$$
(5)

where $P_a(\sigma_a) = \frac{1}{2}(1 + m_a \sigma_a)$ is the probability of the spin σ_a to assume the value σ_a . $P_v(\sigma_a, \sigma_b) = \frac{1}{4}(1 + m_1\sigma_a + m_2\sigma_b + r_v\sigma_a\sigma_b)$ is the pair probability for the spin σ_a , on the sublattice 1, and σ_b , on the sublattice 2, to assume the values σ_a and σ_b , respectively. Analogously, we also define the pair probabilities $P_{hi}(\sigma_a, \sigma_b) = \frac{1}{4}(1 + m_i\sigma_a + m_i\sigma_b + r_{hi}\sigma_a\sigma_b)$ for pairs of first neighbor spins (σ_a, σ_b) on the same sublattice *i*. Defining the following auxiliary quantities:

$$\begin{aligned} x_{1,2} &= \frac{1}{2} (1 + m_{1,2}), \\ y_{1,2} &= \frac{1}{2} (1 - m_{1,2}), \\ v_{1,2} &= \frac{1}{4} (1 \pm m_1 \mp m_2 - r_v), \\ u_{1,2} &= \frac{1}{4} (1 \pm 2m_1 + r_{h1}), \\ t_{1,2} &= \frac{1}{4} (1 \pm 2m_2 + r_{h2}), \\ z_1 &= \frac{1}{4} (1 + m_1 + m_2 + r_v), \\ w_1 &= \frac{1}{4} (1 - m_1 - m_2 + r_v), \\ q_{1,2} &= \frac{1}{4} (1 - r_{h1,h2}), \end{aligned}$$

we can write the equations of motion of interest:

$$\begin{split} \Omega \frac{d}{d\xi} m_{1} &= -m_{1} + \gamma_{1} \left(\frac{u_{1}^{2} z_{1}^{2}}{x_{1}^{3}} + \frac{q_{1}^{2} v_{2}^{2}}{y_{1}^{3}} \right) + 2 \gamma_{2} \left(\frac{u_{1}^{2} z_{1} v_{1}}{x_{1}^{3}} + \frac{q_{1}^{2} w_{1} v_{2}}{y_{1}^{3}} \right) + 2 \gamma_{3} \left(\frac{q_{1} u_{1} z_{1}^{2}}{x_{1}^{3}} + \frac{q_{1} u_{2} v_{2}^{2}}{y_{1}^{3}} \right) + 4 \gamma_{4} \left(\frac{q_{1} u_{1} v_{1} z_{1}}{x_{1}^{3}} + \frac{u_{2} q_{1} v_{2} w_{1}}{y_{1}^{3}} \right) \\ &+ \gamma_{5} \left(\frac{q_{1}^{2} z_{1}^{2}}{x_{1}^{3}} + \frac{u_{2}^{2} v_{2}^{2}}{y_{1}^{3}} \right) + \gamma_{6} \left(\frac{u_{1}^{2} v_{1}^{2}}{x_{1}^{3}} + \frac{q_{1}^{2} w_{1}^{2}}{y_{1}^{3}} \right) + 2 \gamma_{7} \left(\frac{u_{1} v_{1}^{2} q_{1}}{x_{1}^{3}} + \frac{q_{1} u_{2} w_{1}^{2}}{y_{1}^{3}} \right) + 2 \gamma_{8} \left(\frac{q_{1}^{2} z_{1} v_{1}}{x_{1}^{3}} + \frac{u_{2}^{2} w_{1} v_{2}}{y_{1}^{3}} \right) \\ &+ \gamma_{9} \left(\frac{q_{1}^{2} v_{1}^{2}}{x_{1}^{3}} + \frac{u_{2}^{2} w_{1}^{2}}{y_{1}^{3}} \right), \end{split}$$

$$(6)$$

$$\Omega \frac{d}{d\xi} m_{2} = -m_{2} + \gamma_{1} \left(\frac{t_{1}^{2} z_{1}^{2}}{x_{2}^{3}} + \frac{q_{2}^{2} v_{1}^{2}}{y_{2}^{3}} \right) + 2 \gamma_{2} \left(\frac{t_{1}^{2} z_{1} v_{2}}{x_{2}^{3}} + \frac{q_{2}^{2} w_{1} v_{1}}{y_{1}^{3}} \right) + 2 \gamma_{3} \left(\frac{q_{2} t_{1} z_{1}^{2}}{x_{2}^{3}} + \frac{q_{2} t_{2} v_{1} v_{1}}{y_{2}^{3}} \right) \\ &+ \gamma_{5} \left(\frac{q_{2}^{2} z_{1}^{2}}{x_{2}^{3}} + \frac{t_{2}^{2} v_{1}^{2}}{y_{2}^{3}} \right) + 2 \gamma_{7} \left(\frac{t_{1} v_{2} v_{2}}{x_{2}^{3}} + \frac{q_{2} t_{2} v_{1}^{2}}{y_{2}^{3}} \right) + 2 \gamma_{8} \left(\frac{q_{2} t_{1} v_{2} z_{1}}{x_{2}^{3}} + \frac{t_{2} q_{2} v_{1} w_{1}}{y_{2}^{3}} \right) \\ &+ \gamma_{5} \left(\frac{q_{2}^{2} z_{1}^{2}}{x_{2}^{3}} + \frac{t_{2}^{2} v_{1}^{2}}{y_{2}^{3}} \right) + 2 \gamma_{7} \left(\frac{t_{1} v_{2} v_{2} q_{2}}{x_{2}^{3}} + \frac{q_{2} t_{2} v_{1}^{2}}{y_{2}^{3}} \right) + 2 \gamma_{8} \left(\frac{q_{2} z_{1} v_{2} z_{1}}}{x_{2}^{3}} + \frac{t_{2}^{2} w_{1} v_{1}}{y_{2}^{3}} \right) + \gamma_{9} \left(\frac{q_{2}^{2} v_{2}^{2}}{x_{2}^{3}} + \frac{t_{2}^{2} v_{1}^{2}}{y_{2}^{3}} \right) \right) \\ &+ \gamma_{7} \left(\frac{q_{2}^{2} z_{1}^{2}}{x_{2}^{3}} + \frac{t_{2}^{2} v_{1}^{2}}{y_{2}^{3}} \right) + 2 \gamma_{7} \left(\frac{t_{1} v_{2} v_{2}}{x_{2}^{3}} + \frac{q_{2} t_{2} v_{1}^{2}}{y_{2}^{3}} \right) \right) \\ &+ \gamma_{8} \left(\frac{q_{2} v_{1} z_{1}}{x_{2}^{3}} + \frac{t_{2} v_{2} v_{1}^{2}}{y_{2}^{3}} \right) \right) \right)$$

$$\Omega \frac{d}{d\xi} r_{h1} = -2r_{h1} + 2\gamma_1 \left(\frac{u_1^2 z_1^2}{x_1^3} + \frac{q_1^2 v_2^2}{y_1^3} \right) + 4\gamma_2 \left(\frac{u_1^2 v_1 z_1}{x_1^3} + \frac{q_1^2 v_2 w_1}{y_1^3} \right) - 2\gamma_5 \left(\frac{q_1^2 z_1^2}{x_1^3} + \frac{u_2^2 v_2^2}{y_1^3} \right) + 2\gamma_6 \left(\frac{u_1^2 v_1^2}{x_1^3} + \frac{q_1^2 w_1^2}{y_1^3} \right) - 4\gamma_8 \left(\frac{q_1^2 z_1 v_1}{x_1^3} + \frac{u_2^2 v_2 w_1}{y_1^3} \right) - 2\gamma_9 \left(\frac{q_1^2 v_1^2}{x_1^3} + \frac{w_1^2 u_2^2}{y_1^3} \right),$$

$$(8)$$

$$\Omega \frac{d}{d\xi} r_{h2} = -2r_{h2} + 2\gamma_1 \left(\frac{t_1^2 z_1^2}{x_2^3} + \frac{q_2^2 v_1^2}{y_2^3} \right) + 4\gamma_2 \left(\frac{t_1^2 v_2 z_1}{x_2^3} + \frac{q_2^2 v_1 w_1}{y_2^3} \right) - 2\gamma_5 \left(\frac{q_2^2 z_1^2}{x_2^3} + \frac{t_2^2 v_1^2}{y_2^3} \right) + 2\gamma_6 \left(\frac{t_1^2 v_2^2}{x_2^3} + \frac{q_2^2 w_1^2}{y_2^3} \right) - 4\gamma_8 \left(\frac{q_2^2 z_1 v_2}{x_2^3} + \frac{t_2^2 v_1 w_1}{y_2^3} \right) - 2\gamma_9 \left(\frac{q_2^2 v_2^2}{x_2^3} + \frac{w_1^2 t_2^2}{y_2^3} \right),$$
(9)

$$\Omega \frac{d}{d\xi} r_{v} = -2r_{v} + \gamma_{1} \left(\frac{u_{1}^{2} z_{1}^{2}}{x_{1}^{3}} + \frac{q_{1}^{2} v_{2}^{2}}{y_{1}^{3}} + \frac{t_{1}^{2} z_{1}^{2}}{x_{2}^{3}} + \frac{q_{2}^{2} v_{1}^{2}}{y_{2}^{3}} \right) + 2\gamma_{3} \left(\frac{u_{1} q_{1} z_{1}^{2}}{x_{1}^{3}} + \frac{q_{1} u_{2} v_{2}^{2}}{y_{1}^{3}} + \frac{t_{1} q_{2} z_{1}^{2}}{y_{2}^{3}} \right) + 2\gamma_{3} \left(\frac{u_{1} q_{1} z_{1}^{2}}{x_{1}^{3}} + \frac{q_{1} u_{2} v_{2}^{2}}{y_{1}^{3}} + \frac{t_{1} q_{2} z_{1}^{2}}{y_{2}^{3}} \right) + \gamma_{5} \left(\frac{q_{1}^{2} z_{1}^{2}}{x_{1}^{3}} + \frac{q_{2}^{2} z_{1}^{2}}{y_{1}^{3}} + \frac{t_{2}^{2} v_{1}^{2}}{y_{2}^{3}} \right) - \gamma_{6} \left(\frac{u_{1}^{2} v_{1}^{2}}{x_{1}^{3}} + \frac{q_{1}^{2} u_{2}^{2}}{x_{2}^{3}} + \frac{q_{2}^{2} w_{1}^{2}}{y_{2}^{3}} \right) - 2\gamma_{7} \left(\frac{u_{1} q_{1} v_{1}^{2}}{x_{1}^{3}} + \frac{q_{1} u_{2} w_{1}^{2}}{y_{1}^{3}} + \frac{t_{1} q_{2} v_{2}^{2}}{x_{2}^{3}} + \frac{q_{2} t_{2} w_{1}^{2}}{y_{2}^{3}} \right) - \gamma_{9} \left(\frac{q_{1}^{2} v_{1}^{2}}{x_{1}^{3}} + \frac{u_{2}^{2} w_{1}^{2}}{y_{1}^{3}} + \frac{t_{2}^{2} w_{1}^{2}}{y_{2}^{3}} \right),$$
(10)

where

$$\gamma_{1,9} = \tanh\left\{\frac{1}{\theta}\left[\pm 2\pm 2r + h(\xi)\right]\right\},$$
$$\gamma_{2,8} = \tanh\left\{\frac{1}{\theta}\left[\pm 2 + h(\xi)\right]\right\},$$
$$\gamma_{3,7} = \tanh\left\{\frac{1}{\theta}\left[\pm 2r + h(\xi)\right]\right\},$$
$$\gamma_{4} = \tanh\left(\frac{h(\xi)}{\theta}\right),$$
$$\gamma_{5,6} = \tanh\left\{\frac{1}{\theta}\left[\pm 2r + h(\xi)\right]\right\}.$$

We have also defined the following reduced variables: $\theta = k_B T/J_1$, $r = J_2/J_1$, $h = h_0 \cos \xi$, $\Omega = w\tau$, $\xi = wt$, and $h_0 = H_0/J_1$. In order to characterize the ferromagnetic and antiferromagnetic states we have defined the order parameters $m_f = (m_1 + m_2)/2$ and $m_a = (m_1 - m_2)/2$, respectively.

We have shown that for $H_0=0$, the phase diagram of this model in the plane temperature versus r exhibits the ferromagnetic, antiferromagnetic, and paramagnetic phases [12]. On the other hand, for a constant magnetic field and r>0, the phase diagram in the plane (h_0, θ) displays a tricritical point [8] for any value of the competing ratio r.

Here we are interested in the behavior of the model when the magnetic field *H* is varying sinusoidally in time. In this case, we look for the nonequilibrium stationary states of the system. The set of Eqs. (6)–(10) is solved numerically by using the Runge-Kutta method of fourth order for fixed values of θ , r>0, and Ω . Depending on the value assumed by h_0 , the antiferromagnetic order parameter m_a can oscillate around a given nonzero mean value or it becomes zero. But in any case the ferromagnetic order parameter m_f always oscillates around a zero mean value. When the value of m_a is identical to zero anytime, the sublattice magnetizations m_1 and m_2 are the same time functions. In this case the solution is called symmetric or paramagnetic. It occurs for high values of both temperature and amplitude of oscillating field. For lower values of the amplitude h_0 , the symmetric solution becomes unstable and a new behavior is observed. The solution m_a oscillates, and its mean value during a period of oscillation of the magnetic field is different from zero. This is the so-called antiferromagnetic or nonsymmetric solution. It is interesting to define a dynamical order parameter for systems subject to external periodic forces, as being the mean value of their corresponding instantaneous values over a cycle of oscillation. In this way, for the dynamical ferromagnetic and antiferromagnetic order parameters M_f and M_a , respectively, we write



FIG. 1. Ferromagnetic order parameter M_f and Lyapunov exponents λ_f^s and λ_f^n in a continuous transition. We have taken r = -1.0, $\Omega/2\pi = 0.1$, and $\theta = 0.55$. h_0^c indicates the amplitude of the critical field.



FIG. 2. The same legend as in Fig. 1, except that the transition is discontinuous, and we have a coexistence of the paramagnetic and ferromagnetic phases between the zeros of the Lyapunov exponents.

$$M_{a,f} = \frac{1}{2\pi} \int_0^{2\pi} m_{a,f} d\xi.$$
 (11)

In order to analyze the stability of the symmetric and nonsymmetric solutions we have calculated the corresponding Lyapunov exponents [2], which can be obtained by the following equation:

$$\Omega\lambda_{a,f} = \frac{1}{2\pi} \int_0^{2\pi} \frac{\partial G_{a,f}}{\partial m_{a,f}} d\xi, \qquad (12)$$

where the functions $G_{a,f}$ are obtained as one-half of the difference or sum of the right-hand sides of Eqs. (6) and (7), respectively.

In the following we present the main results of this work. In Fig. 1 we exhibit for the ferromagnetic case, with r=-1.0, and for the values $\theta=0.55$ and $\Omega/2\pi=0.1$, the dynamical ferromagnetic order parameter M_f as a function of the amplitude of the oscillating magnetic field. For this selected set of parameters we see that M_f goes continuously to zero at the critical value of the field h_0^c . In the same figure we also exhibit the behavior of the corresponding symmetrical, λ_f^s , and of the nonsymmetrical, λ_f^n , Lyapunov exponents as a function of the amplitude of the field. In this case



FIG. 3. Dynamical phase diagram in the plane amplitude of the oscillating field versus reduced temperature for r = -1.0 and $\Omega/2\pi = 0.1$. *P* and *F* represent the paramagnetic and ferromagnetic phases, respectively, while the *P*+*F* region indicates the coexistence between the *P* and *F* phases. θ_t gives the temperature of the dynamical tricritical point.



FIG. 4. Dynamical phase diagram in the plane reduced amplitude of the oscillating field versus reduced temperature for r=1.0and $\Omega/2\pi=0.1$. *P* and AF represent the paramagnetic and antiferromagnetic phases, respectively, while the *P*+AF region indicates the coexistence between the *P* and AF phases. θ_t gives the temperature of the dynamical tricritical point.

of continuous transition we see that both exponents are always negative, and go to zero at exactly the same value of the critical field. In Fig. 2, and still for r = -1.0, $\theta = 0.40$, and $\Omega/2\pi = 0.1$, we observe a different behavior of the order parameter. Now the transition is discontinuous, and the corresponding Lyapunov exponents become zero at different values of the field. This characterizes a region of the coexistence of the symmetrical and nonsymmetrical solutions. Physically, for the values of the field in between the zeros of the Lyapunov exponents we have a coexistence of the ferromagnetic and paramagnetic phases. In Fig. 3 we present the complete phase diagram in the plane amplitude of the field versus reduced temperature. For r and Ω we use the same parameters of Figs. 1 and 2. For $\theta > \theta_t$ the transition between the ferromagnetic and paramagnetic phases is continuous, like that in Fig. 1, while for $\theta < \theta_t$ we have a coexistence of these two phases, as shown in Fig. 2. The temperature θ_t is the so-called dynamical tricritical temperature. The phase diagram we have obtained for this model in the dynamical pair approximation is similar to the one obtained by Tomé and de Oliveira [2] in the mean-field approach, and confirmed by Monte Carlo simulations [13,14].

Now we turn to the most interesting case of competition between the exchange couplings. In this case the model is a proper layered metamagnetic model. Here, the dynamical order parameter of interest is M_a , which accounts for the an-



FIG. 5. The same legend as in Fig. 4, but here r = 10.0. The tricritical point disappears and we have only continuous transitions between the AF and *P* phases.



FIG. 6. Critical amplitude of the oscillating field as a function of the ratio *r*, at zero temperature. Curves 1 and 2 give the limits of stability of the AF and *P* phases, respectively. Here $\Omega/2\pi=0.1$.

tiferromagnetic coupling between the layers of the model. For instance, in Fig. 4 we show the complete dynamical phase diagram for r=1.0 and $\Omega/2\pi=0.1$. For these values the diagram is topologically similar to the pure ferromagnetic case, Fig. 3, except that the ordered phase is the antiferromagnetic one. For the construction of this phase diagram we have calculated all the Lyapunov exponents for the symmetrical and nonsymmetrical solutions. Although the absolute values of r are the same as in Figs. 3 and 4, we observe that the temperature of the dynamical tricritical point θ_t is slightly different in the metamagnetic model. This is because in the pair approximation we are using, the correlation functions r_{h1} and r_{h2} for the sublattices 1 and 2 are distinct. In Fig. 5, we take r = 10.0 and $\Omega/2\pi = 0.1$. For this value of the competition parameter the dynamical tricritical point disappears. This is expected because as we increase the values of r the model becomes almost antiferromagnetic in nature, and in this case the transition line is continuous [3]. In Fig. 6 we show the plot of the amplitude of the critical field for the limits of stability of the antiferromagnetic phase, curve 1, and of the paramagnetic phase, curve 2, at zero temperature, and for $\Omega/2\pi = 0.1$, as a function of r. It is clear that for r > 5.8, there is no longer coexistence of phases, even at zero temperature, and we observe only a continuous phase transition between the antiferromagnetic and paramagnetic phases. We also observe that when r goes to zero, the layers become uncoupled, and we have only a collection of onedimensional ferromagnetic Ising models. The amplitude of the critical oscillating field is $h_0^c = 2.0J_1$, and the limits of stability of the antiferromagnetic and paramagnetic phases



FIG. 7. The same legend as in Fig. 6 but now we are using a small value for the frequency $\Omega/2\pi = 0.01$.

coincide. The behavior shown in the latter figure depends on the frequency of the field. For instance, we show in Fig. 7 a plot similar to that of Fig. 6, except that now $\Omega/2\pi=0.01$. In this limit the field is almost static, and we always have a phase coexistence at zero temperature for any value of $r \neq 0$. The difference between the limits of stability of the critical fields remains constant for very small values of the frequency of the external oscillating field.

In summary, we have studied the dynamical behavior of an Ising model in a square lattice, with competing horizontal and vertical exchange interactions, in the presence of a periodic oscillating magnetic field. We have found the stationary states of the model through the master equation approach and within the dynamical pair approximation. The phase diagram of the model in the plane amplitude of the critical field versus temperature was determined for the ferromagnetic and layered metamagnetic models, for different values of the frequency of the field and of the competition parameter. The stability of the continuous and discontinuous transitions was analyzed in terms of the appropriate Lyapunov exponents for the symmetrical and nonsymmetrical solutions. We have seen that for the metamagnetic model the dynamical tricritical point disappears for large values of the competition parameter. On the other hand, for very slowly varying fields, the difference between the amplitudes of the critical fields for the limits of stability of the antiferromagnetic and paramagnetic phases is constant for almost all values of the competition parameter.

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